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Structural evolution of the one-dimensional spectral function from the low- to the high-energy limit

Hideaki Maebashi and Yasutami Takada
Institute for Solid State Physics, University of Tokyo, Kashiwa, Chiba 277-8581, Japan
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By exactly analyzing the spin-$\frac{1}{2}$ Luttinger liquid (LL) and numerically solving a model of a mobile impurity electron in the LL, we obtain the one-electron spectral function $A(p,\omega)$ in a one-dimensional metal in an entire range of p at zero temperature. For $|p|$ near the Fermi point $p_{\rm F}$, $A(p,\omega)$ is featured by two prominent peaks of spinon and (anti)holon representing spin-charge separation, but we also find an additional cusp structure between them. For $|p| \gg p_{\rm F}$, this structure evolves as a main peak in $A(p,\omega)$ by swallowing the antiholon mode and its dispersion relation approaches that of a free electron, implying the existence of an electron excitation in the whole region, but not quite a quasiparticle in the Fermi liquid due to ever existing power-law decay of the excitation.

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The concept of spin-charge separation plays a central role in describing low-energy physics near Fermi points in a one-dimensional (1D) interacting electron gas, a typical example of the spin-$\frac{1}{2}$ Luttinger liquid (LL) [1]. This concept may be confirmed in real materials by various experiments [2], including the recent high-resolution angular resolved photoemission spectroscopy in which the one-electron spectral function $A(p,\omega)$ can be directly measured in the wide range of momentum p and energy ω.

If $|p|$ is not restricted to the region near the Fermi momentum $p_{\rm F}$, the linear spectrum approximation, usually adopted in the LL theory, is not sufficient in appropriately obtaining $A(p,\omega)$. In fact, the effect of the nonlinear spectrum on $A(p,\omega)$ has been intensively studied in recent years [3–8]. According to those studies on integrable systems, $A(p,\omega)$ has singularities for arbitrary p as $A(p,\omega) \propto |\omega - \epsilon_\nu(p)|^{-\mu_\nu(p)}$ with $\nu = s$ and c, where $\epsilon_s(p)$ and $\epsilon_c(p)$ are energies of spin and charge collective excitations, respectively. In the usual LL theory, the exponent $\mu_\nu(p)$ is independent of p, but the nonlinearity in the electron dispersion makes it depend on p [5–9]. Since the edge of support of $A(p,\omega)$ is located at $\omega = \epsilon_s(p)$, $\mu_s(p)$ determines the power of the threshold singularity in $A(p,\omega)$ and its actual value has been given from the finite-size spectrum obtained by the Bethe-ansatz method [6,8]. For nonintegrable systems, this threshold singularity remains intact, but the singularity at $\omega = \epsilon_c(p)$ is smeared into a broad peak [7].

In those preceding works, only the singularities at $\omega = \epsilon_s(p)$ and $\epsilon_c(p)$ are discussed on the belief that the electron nature will not sustain in the spin-charge separated system. For $|p|$ far away from $p_{\rm F}$, however, the effect of interactions becomes so weak that we would naively expect that the nature of an injected electron to measure $A(p,\omega)$ manifests itself as a main peak in $A(p,\omega)$. Then a natural question arises: *Does an electronlike excitation mode actually exist in the 1D interacting electron gas for* $|p| \gg p_{\rm F}$*?* If yes, a related and more intriguing question is, *How does the electronlike mode reconcile with the physics of spin-charge separation for* $|p| \approx p_{\rm F}$*?*

In this Rapid Communication, we have carefully studied the 1D one-electron Green's function $G(p,t)$ in momentum space and time and found that for $p \approx p_{\rm F}$, its long-time asymptotic form is composed of *three* independent modes of power-law decay. Two of them correspond to well-known spinon and (anti)holon excitations, but the remaining one describes the mode of an electronlike particle (*pseudoelectron*) which may be regarded as an electron dressed with a "cloud" of low-lying spin and charge collective excitations. This pseudoelectron does not appear as a main structure in $A(p,\omega)$ and never leads to a finite jump in the momentum distribution function $n(p)$. As p goes away from $p_{\rm F}$, the pseudoelectron structure gets broader, but with the further increase of p, it becomes less broad and eventually for $p \gg p_{\rm F}$, it evolves as a main and divergent peak in $A(p,\omega)$ by swallowing the antiholon mode. Concomitantly, its dispersion relation approaches that of a free electron, allowing us to regard the pseudoelectron as a free electron; but actually it is not quite, nor the Landau's quasiparticle, basically because this excitation is accompanied by power-law decay. Those results clarify the generic feature of $A(p,\omega)$ in a 1D metal and answer the aforementioned two questions. In the following, we shall substantiate our claim.

Let us consider the spin-$\frac{1}{2}$ Luttinger model, for which $G(p,t)$ with $p = p_{\rm F} + k$ is well known and is given by [10,11]

$$iG(p_{\rm F}+k,t) = \int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{e^{-ikx}}{\eta - i(x - v_{\rm F}t)} \frac{1 - i\Lambda(x - v_{\rm F}t)}{\sqrt{1 - i\Lambda(x - u_s t)}} \times [1 - i\Lambda(x - u_c t)]^{-(1+\theta)/2} \times [1 + i\Lambda(x + u_c t)]^{-\theta/2}, \quad (1)$$

for $t > 0$ with η a positive infinitesimal and Λ a finite momentum-transfer cutoff. Here u_s and u_c are the velocities of spinon and (anti)holon, respectively, and θ denotes the power of singularity in $n(p)$. They are related to the velocity of an electron $v_{\rm F}$ and the interactions between electrons g_4 and g_2 in the original Hamiltonian through $u_s = v_{\rm F} - g_4/2\pi$, $u_c = [(v_{\rm F} + g_4/2\pi)^2 - (g_2/\pi)^2]^{1/2}$, and $\theta = (v_{\rm F} + g_4/2\pi - u_c)/2u_c$ [12]. By eliminating g_4 from those equations, we obtain $v_{\rm F} = [(1 + 2\theta)u_c + u_s]/2$, and thus $v_{\rm F}$ as well as u_s, u_c, and θ can be determined by the Bethe-ansatz method for integrable systems.

By regarding the integrand in Eq. (1) as an analytic function of x to deform the integral path along the real axis into the lower-half complex x plane and assuming $u_s < v_{\rm F} < u_c$, we can evaluate $iG(p_{\rm F}+k,t)$ for $k > 0$ in the long-time limit of $t \to \infty$ as

$$iG(p_{\rm F}+k,t) = e^{-k/\Lambda} \sum_{\nu=s,c} \left(\frac{\Lambda}{k}\right)^{1/2-\gamma_\nu} \frac{e^{-iu_\nu kt + i\pi\phi_\nu^{\rm LL}/2}}{\left(\Omega_\nu^{\rm LL} t\right)^{1-\mu_\nu^{\rm LL}}} + (\Omega_{\rm F} t)^{-1+\mu_{\rm F}} e^{-iv_{\rm F}kt + i\pi\phi_{\rm F}/2}. \quad (2)$$

The terms specified by $\nu=s,c$ represent the contributions from contours along the two branch cuts (which we take parallel to the imaginary axis) associated with the branch points at $x=u_s t-i/\Lambda$ and $u_c t-i/\Lambda$, showing the well-known time correlation specific to spinon and (anti)holon in the LL with the exponents $\mu_s^{\rm LL}$ and $\mu_c^{\rm LL}$ given, respectively, as $\mu_s^{\rm LL}=1+\phi_s^{\rm LL}=1/2-\theta$ and $\mu_c^{\rm LL}=\phi_c^{\rm LL}=(1-\theta)/2$. Here $\gamma_s=0$, $\gamma_c=\theta/2$, $\Omega_s^{\rm LL}=[(u_c+u_s)^{\theta/2}(u_c-u_s)^{(1+\theta)/2}\sqrt{\pi}]^{2/(1+2\theta)}\Lambda$, and $\Omega_c^{\rm LL}=[(2u_c)^{\theta/2}(u_c-u_s)^{1/2}\Gamma(\frac{1+\theta}{2})]^{2/(1+\theta)}\Lambda$ with $\Gamma(z)$ the gamma function. The last term denotes the contribution from a simple pole at $x=v_{\rm F}t-i\eta$, seemingly describing an electron moving with the velocity $v_{\rm F}$, but not quite a usual quasiparticle in the Fermi liquid owing to the existence of the power-law decay with the exponent $\mu_{\rm F}(\neq 1)$, where $\mu_{\rm F}=\phi_{\rm F}=-\theta$ and $\Omega_{\rm F}=[(u_c-v_{\rm F})^{(1+\theta)/2}(u_c+v_{\rm F})^{\theta/2}(v_{\rm F}-u_s)^{1/2}]^{1/(1+\theta)}\Lambda$. Thus we shall call it a *pseudoelectron*. Its decaying behavior persists even at $k\to 0$.

Casting $iG(p,t)$ in Eq. (2) into the form

$$iG(p,t)=\sum_{\ell=s,c,e}[\Omega_\ell(p)t]^{-1+\mu_\ell(p)}e^{-i\epsilon_\ell(p)t+i\pi\phi_\ell(p)/2}, \quad (3)$$

and evaluating $A(p,\omega)$ by $\pi^{-1}{\rm Re}\int_0^\infty dt\, e^{i\omega t}iG(p,t)$ for $\omega>0$, we see that $A(p,\omega)$ has a singularity (peak or shoulder) at $\omega=\epsilon_\ell(p)$ for $|\mu_\ell(p)|<1$ in the way of

$$A(p,\omega)={\rm const}+C_\ell^{\pm}(p)|\omega-\epsilon_\ell(p)|^{-\mu_\ell(p)}, \quad (4)$$

where $C_\ell^{\pm}(p)=\pi^{-1}\Omega_\ell(p)^{-1+\mu_\ell(p)}\Gamma[\mu_\ell(p)]\cos(\pi[\mu_\ell(p)\pm\phi_\ell(p)]/2)$ with the upper (lower) sign for ω larger (smaller) than $\epsilon_\ell(p)$.

As for the term $\ell=e$ in Eq. (3) in which $\epsilon_e(p_{\rm F}+k)=v_{\rm F}k$, $\mu_e(p_{\rm F})=\mu_{\rm F}$, and $\phi_e(p_{\rm F})=\phi_{\rm F}$ with $\Omega_e(p_{\rm F})=\Omega_{\rm F}$ for the spin-$\frac{1}{2}$ Luttinger model, we easily see that $\partial A(p_{\rm F}+k,\omega)/\partial\omega$ diverges at $\omega=v_{\rm F}k$ for $0<\theta\leqslant 1/8$ even at $p\to p_{\rm F}$. By using the identity $\int_0^\infty t^{\mu_{\rm F}}e^{\pm it}dt=\Gamma(\mu_{\rm F}+1)e^{\pm i\pi(\mu_{\rm F}+1)/2}$, we can also verify that $A(p,\omega)$ behaves in accord with Eq. (4), exhibiting a structure of the pseudoelectron.

In Fig. 1, we have explicitly shown the generic feature of $A(p,\omega)$ in which a peak with a cusp exists at the pseudoelectron mode $[\omega=\epsilon_e(p)]$ in addition to the well-known double divergent peaks for p near $p_{\rm F}$. In plotting $A(p,\omega)$, we need to know concrete values for u_s, u_c, and θ; we have determined them by adopting the Yang-Gaudin model [13], i.e., the 1D electron gas with a δ-function interaction, described by the Hamiltonian $H=-\frac{1}{2m}\sum_{i=1}^N\frac{\partial^2}{\partial x_i^2}+V_0\sum_{i<j}\delta(x_i-x_j)$, in the intermediate-coupling region of $\lambda_0\equiv mV_0/2\pi p_{\rm F}=1.0$.

Two comments are in order on the result in Fig. 1: (i) Irrespective of Λ, the limit of $\eta\to 0^+$ is required in Eq. (1) to keep the correct electron anticommutation relation $\{c_{p\sigma},c_{p'\sigma'}^\dagger\}=\delta_{pp'}\delta_{\sigma\sigma'}$ so as to satisfy the sum rule $\int_{-\infty}^\infty A(p,\omega)d\omega=\langle\{c_{p\sigma},c_{p\sigma}^\dagger\}\rangle=1$ [14]. Thus the proper limiting procedure in Eq. (1) is to make η zero first with Λ being finite, leading to the result in Fig. 1. If η is replaced by Λ^{-1} as was previously the case, the pole contribution disappears, because the residue vanishes with that choice of η, constituting the reason for the omission of the pseudoelectron contribution in preceding studies [15,16]. That choice is physically incorrect, because the anticommutation relation is not globally satisfied, violating the sum rule [17]. (ii) If we take $\theta=0$ in Eq. (1), essential

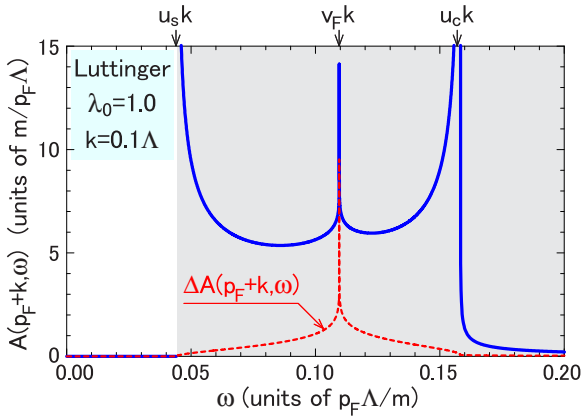

FIG. 1. (Color online) Full spectral function $A(p_{\rm F}+k,\omega)$ at $k=0.1\Lambda$ for the spin-$\frac{1}{2}$ Luttinger model with use of $u_s=0.4406p_{\rm F}/m$, $u_c=1.582p_{\rm F}/m$, and $\theta=0.05\,351$ obtained by the Bethe ansatz for the Yang-Gaudin model with $\lambda_0=1.0$ (solid curve). The contribution from a pseudoelectron excitation ($\omega=v_{\rm F}k$), which has been omitted in preceding studies [15,16], is explicitly shown by $\Delta A(p_{\rm F}+k,\omega)$ (dotted curve).

simplification occurs, leading to an analytic expression of $A(p,\omega)$ in terms of the confluent hypergeometric function of two variables. In this case, Voit found that $A(p,\omega)$ has an additional peak with logarithmic singularity at $\omega=v_{\rm F}k$ besides the spinon and holon ones, while $n(p)$ remains the same as the noninteracting one with a finite jump at $|p|=p_{\rm F}$ [18]. But, he gave up calculations without the replacement of η by Λ^{-1} for the spin-$\frac{1}{2}$ Luttinger model with $\theta\neq 0$ because of the difficulty in dealing with a double Fourier transform in $A(p,\omega)$. We have overcome this difficulty by use of a mathematical trick virtually identical to Feynman parameters [16,19]. By introducing two Feynman parameters, we can analytically perform the double Fourier transform and then resort to numerical computation of the remaining double Feynman integral. The result thus computed with high accuracy is plotted in Fig. 1 and exhibits a singular behavior precisely matched up with that suggested in Eq. (4).

Now let us discuss the effect of nonlinear dispersion on $A(p,\omega)$ as well as the case of $|p|$ far away from $p_{\rm F}$ by adopting the mobile impurity model which was previously introduced for treating spinon and holon modes without spoiling the rigor of the whole theory [5,7]. Our strategy is to employ the same model to calculate the contribution to $A(p,\omega)$ from the pseudoelectron excitation. In the model, we first consider an *electron* with either linear or nonlinear dispersion ξ_p as a mobile impurity in the LL with the velocity $v_p=\partial\xi_p/\partial p$. (Here we take only the case of $\xi_p\geqslant 0$, i.e., $|p|\geqslant p_{\rm F}$.) This impurity electron is assumed to couple with spin and charge collective excitations near the right (+) and left (−) Fermi points with the interaction $\tilde{V}_{\alpha\nu}\equiv\tilde{V}_{\alpha\nu}(p)$ ($\alpha=\pm$, $\nu=s,c$) [20]. Then, we can calculate the Green's function for the mobile impurity electron by straightforwardly extending the method developed by Dzyaloshinskiĭ and Larkin [21,22] or in a more general GWΓ framework [23,24], from which we obtain an integral equation to determine the Green's function $G(p+k,\omega)$

in momentum-frequency space as

$$(\omega-\xi_p-v_pk)G(p+k,\omega) = 1+i\int\frac{dq\,d\varepsilon}{(2\pi)^2}\frac{W(q,\varepsilon)}{\varepsilon-v_pq}G(p+k-q,\omega-\varepsilon)e^{-|q|/\Lambda}, \tag{5}$$

with Λ the cutoff of momentum transfer q and $W(q,\varepsilon)=(1/4)\sum_{\alpha\beta\nu}\tilde{V}_{\alpha\nu}\chi_{\alpha\beta\nu}(q,\varepsilon)\tilde{V}_{\beta\nu}$ with use of $\chi_{\alpha\beta\nu}(q,\varepsilon)$ the exact correlation functions for the Luttinger model.

Since Eq. (5) is very similar to the one known well in the Luttinger model [10], we are familiar with its solution; by transforming the variables from momentum and frequency into real space and time, we can write down a differential equation for $G(x,t)$, from which we obtain

$$G(p+k,t)=\frac{-ie^{-i\epsilon_e(p+k)t}}{\prod_{\nu=c,s}\prod_{\alpha=\pm}[1+i(u_\nu-\alpha v_p)\Lambda t]^{(\delta_{\alpha\nu}/2\pi)^2/2}}, \tag{6}$$

for $t>0$, where $\epsilon_e(p+k)$, the dressed impurity energy, contains an energy shift in proportion to the cutoff Λ as

$$\epsilon_e(p+k)=\xi_p+v_pk-\Lambda\sum_{\alpha\nu}(\delta_{\alpha\nu}/2\pi)^2(u_\nu-\alpha v_p)/2. \tag{7}$$

In Eqs. (6) and (7), phase shifts $\delta_{\alpha\nu}\equiv\delta_{\alpha\nu}(p)$ are *independent* of Λ and related to $\tilde{V}_{\alpha\nu}$ and v_p through

$$\delta_{\pm c}=\frac{\tilde{V}_{\pm c}\sqrt{1+\theta}-\tilde{V}_{\mp c}\sqrt{\theta}}{u_c\mp v_p},\quad \delta_{\pm s}=\frac{\tilde{V}_{\pm s}}{u_s\mp v_p}. \tag{8}$$

The results in Eqs. (7) and (8) indicate that $\epsilon_e(p_F+k)=v_Fk$, $\delta_{+c}(p_F)=2\pi\sqrt{1+\theta}$, $\delta_{-c}(p_F)=2\pi\sqrt{\theta}$, $\delta_{+s}(p_F)=2\pi$, and $\delta_{-s}(p_F)=0$. Thus at $p=p_F$, we see that Eq. (6) is reduced to the pole contribution in Eq. (1).

For illustration of the overall behavior of $A(p>p_F,\omega)$ with the change of p and ω, we adopt the Yang-Gaudin model in the weak-coupling region ($\lambda_0\leqslant 0.1$) in which analytic expressions such as $v_F=p_F/m$, $u_s=(1-\lambda_0)v_F$, $u_c=\sqrt{1+2\lambda_0}v_F$, and $\theta=(\sqrt{1+2\lambda_0}-1)^2/4\sqrt{1+2\lambda_0}$ are successfully checked to reproduce the exact Bethe-ansatz results with sufficient accuracy, indicating that we may well specify the mobile impurity model by employing the weak-coupling results of the quadratic dispersion $\xi_p=(p^2-p_F^2)/2m$ and the interactions $\tilde{V}_{\pm c}(p)=-\tilde{V}_{+s}(p)=V_0$ and $\tilde{V}_{-s}(p)=V_0/(1-2\lambda_0\ln[\xi_p/E_i])$, all of which are obtained by the poor man's scaling with E_i, an initial energy scale. Then, from $G(p,t)$ in Eq. (6), we can explicitly compute $A(p,\omega)$ which is insensitive to the choice of E_i. In Fig. 2, $A(p,\omega)$ thus obtained at $\lambda_0=0.1$ is displayed with increasing p from p_F to show its complete structural evolution in the 1D weakly interacting electron gas with the quadratic dispersion. Since we focus on the region of ω in the very vicinity of ξ_p, only the pseudoelectron mode appears as a singular structure in $A(p,\omega)$ in Fig. 2.

By comparing Eq. (6) in its long-time limit with Eq. (3), we can determine $\mu_e(p)$ and $\phi_e(p)$ as

$$\mu_e(p)=1-\sum_{\alpha\nu}(\delta_{\alpha\nu}/2\pi)^2/2, \tag{9}$$

$$\phi_e(p)=-\sum_{\alpha\nu}(\delta_{\alpha\nu}/2\pi)^2\mathrm{sgn}(u_\nu-\alpha v_p)/2. \tag{10}$$

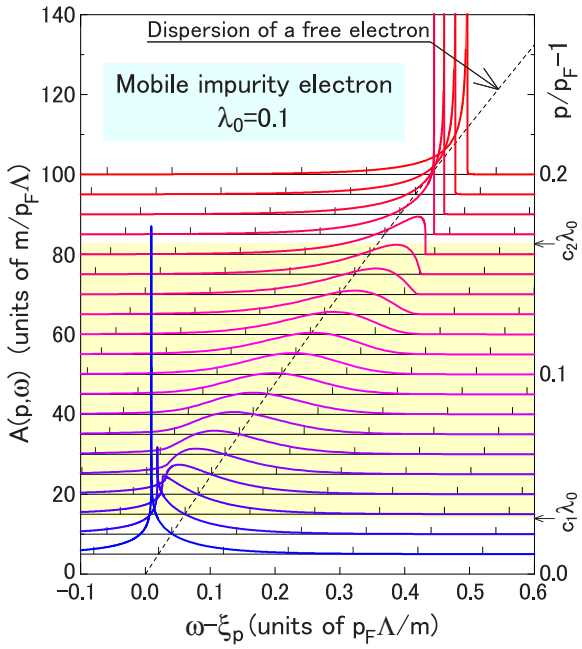

FIG. 2. (Color online) Structural evolution of $A(p,\omega)$ as a function of ω in the very vicinity of ξ_p with p going away from p_F (from $p/p_F-1=0.01$ to $p/p_F-1=0.2$ by the interval 0.01) in the mobile impurity model at $\lambda_0=0.1$.

They are independent of Λ and universal, but they depend on p in the system with nonlinear dispersion [9], as plotted in Fig. 3 in which $\mu_e(p)$ and $|\phi_e(p)|-1$ are given by the solid and dotted curves, respectively. The structural feature of $A(p,\omega)$ is specified by both $\mu_e(p)$ itself and its relative value with respect to $|\phi_e(p)|-1$. Thus we define two dimensionless constants, $c_1=1-1/\sqrt{2}+O(\lambda_0)$ and $c_2=\sqrt{3}+O(\lambda_0)$, corresponding to the intersections of the solid and dotted curves. By virtue of Eq. (4), $A(p,\omega)$ at $\omega=\epsilon_e(p)$ has a peak with a cusp for $p/p_F-1<c_1\lambda_0$ (the spin-charge separated regime) and a divergent one for $p/p_F-1>c_2\lambda_0$ except for the narrow vicinity of $p/p_F-1=c_2\lambda_0$ (the nearly-free-electron regime). In the intermediate regime of $c_1\lambda_0<p/p_F-1<c_2\lambda_0$, on the

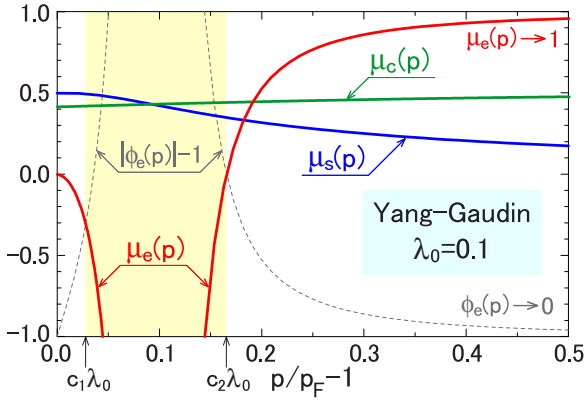

FIG. 3. (Color online) Leading exponents of the spectral function at the spinon, antiholon, and pseudoelectron modes, which are denoted by $\mu_s(p)$, $\mu_c(p)$, and $\mu_e(p)$, respectively, versus p/p_F-1 for the Yang-Gaudin model with $\lambda_0=0.1$.

other hand, $A(p,\omega)$ shows only a broad peak around $\omega=\xi_p$ due to the strong damping effect of $\delta_{+c}\to\pm\infty$ for $v_p\to u_c$ in Eq. (8) (the pseudoelectron damping regime).

In order to give an entire picture of $A(p,\omega)$ including spinon and antiholon modes, we have also plotted the exponents $\mu_s(p)$ and $\mu_c(p)$ in Fig. 3, which are exact for the Yang-Gaudin model with $\lambda_0=0.1$. (Accurate values for those exponents can be derived from the exact Bethe-ansatz results for $\epsilon_s(p)$ and $\epsilon_c(p)$ for Galilean invariant systems [7].) By comparing them with $\mu_e(p)$, we can identify the mode(s) to dominate $A(p,\omega)$: (i) For $|p|/p_F-1\ll\lambda_0$, the situation is characterized by $\mu_s(p)\approx\mu_c(p)\gg\mu_e(p)$, implying the dominance of spinon and antiholon modes. Thus the physics here is well described by the concept of spin-charge separation, but the pseudoelectron is now found to appear as an additional singular cusp structure as depicted in Fig. 1. (ii) For $|p|/p_F-1\sim\lambda_0$, the pseudoelectron excitation is overdamped as shown in Fig. 2 and sandwiched between the spinon and antiholon divergent peaks, making it difficult to be detected, although we do not expect that its total contribution to the spectral weight is negligible. (iii) For $|p|/p_F-1\gg\lambda_0$, $\mu_e(p)$ becomes much larger than either $\mu_s(p)$ or $\mu_c(p)$, indicating that the long-time evolution of $G(p,t)$ is controlled by the single mode of pseudoelectron, well defining the nearly-free-electron regime. In fact, according to our explicit calculation based on Ref. [7], $\mu_s(p)$ is a decreasing function of $|p|$ and becomes negative for $|p|\gtrsim 2p_F$, so that the divergence at the edge of support of $A(p,\omega)$ disappears. As for the antiholon, $\mu_c(p)$ slowly increases with increasing $|p|$ and reaches $1/2$ at $|p|\to\infty$. In the high-$|p|$ limit with λ_0 fixed, both $\epsilon_e(p)$ and $\epsilon_c(p)$ approach the free-electron dispersion, but because $\mu_e(p)\approx 2\mu_c(p)\approx 1$, the antiholon peak gets absorbed into the pseudoelectron one. Thus $A(p,\omega)$ is composed of a single divergent peak, reduced to a delta function at $|p|\to\infty$ as $A(p,\omega)\approx(v_p\Lambda)^{-1}f_\gamma([\omega-p^2/2m]/v_p\Lambda)$ where $f_\gamma(y)\equiv(2\pi)^{-1}\int_{-\infty}^{\infty}ds(1+s^2)^{-\gamma}e^{iys}\approx\gamma|y|^{-1+2\gamma}$ with $\gamma=[1-\mu_e(p)]/2\propto(p_F/p)^2\to 0^+$.

Our discussion has been limited only for $|p|\geqslant p_F$, but a similar discussion can be made for $|p|<p_F$ in which electron-hole asymmetry due to nonlinearity in the dispersion may be a matter of interest. Our theory is applicable to lattice systems by suitably choosing the dispersion ξ_p and an interaction parameter λ_0. Then the pseudoelectron damping regime appears in general for $|1-|v_p|/v_F|\sim\min(1,\lambda_0)$ due to the strong scattering effect for $|v_p|=|\partial\xi_p/\partial p|\to u_c,u_s$. According to the numerical results for $A(p,\omega<0)$ obtained by the dynamical density-matrix renormalization group (DDMRG) method for the 1D Hubbard model in the intermediate-coupling region of $U=4.9t$ and the filling $n=0.6$ (for which $\lambda_0\sim 1$) [25], the peaks of spinon, holon, and its shadow band for $|p|<p_F$, which are predicted by the Bethe ansatz, are found, but there appears no signature of an additional noticeable structure, at least for $|p|$ not close to p_F. This absence of the pseudoelectron structure in the DDMRG does not contradict our theory, because the data without a strong artificial broadening effect are given only in the pseudoelectron damping regime of $1-|v_p|/v_F\sim\lambda_0$.

In summary, we have theoretically studied the overall behavior of $A(p,\omega)$ in a 1D metal at zero temperature in order to establish the concept of a "pseudoelectron," describing the behavior of an electron injected into the LL with either linear or nonlinear dispersion. This pseudoelectron is found to manifest itself in an entire range of p, though its importance in the whole structure of $A(p,\omega)$ depends on p; for $|p|\approx p_F$, it appears only as an additional cusp structure to the main peaks of spinon and (anti)holon, while for $|p|\gg p_F$, it provides a main and divergent peak with the correct high-$|p|$ limit of a delta function. This pseudoelectron very much resembles a quasiparticle in higher-dimensional Fermi-liquid systems (e.g., see Fig. 3 in Ref. [24]), although it is not quite the same, reflecting the specialty of 1D physics. We hope that this concept of a pesudoelectron will be confirmed in the future through experiment and/or large-scale numerical calculation with deliberately chosen parameters so as to avoid its overdamping regime.

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