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ORIGINAL PAPER

Generic Features of an Electron Injected into the Luttinger Liquid

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Abstract An extra electron injected into the Luttinger liquid (LL) is not completely separated into spin and charge excitations, but it provides a peak of an electron-like excitation in addition to the two prominent spin- and charge-excitation peaks in the one-electron spectral function of an interacting one-dimensional (1D) system. By a scaling argument based on a renormalization-group scheme, we obtain universal features of this electron-like excitation for low energies and in the vicinity of the Fermi point, which are entirely characterized by the Luttinger parameter K_c and the renormalized Fermi velocity $v_{\rm F}$ at the LL fixed point. The latter quantity $v_{\rm F}$ is shown to be given by the slope of the lowest excitation spectrum near the Fermi point in the system with large but finite size. Some of exact results are presented for the 1D Hubbard model at $n = 0.59$ filling.

Keywords One-dimensional electron systems · Luttinger liquid · Spin-charge separation · One-electron spectral function

1 Introduction

A Luttinger liquid (LL) is the most typical example of non-Fermi liquids that are characterized by the absence of a discontinuity in the momentum distribution function $n(k)$ at temperature $T = 0$. It describes universal features of an interacting one-dimensional (1D) electron system for low energies, long wavelengths, and/or in the vicinity of the Fermi point $k_{\rm F}$ [1–3]. One of those features is a power-law singularity. For example, the behavior of $n(k)$ with momentum k near $k_{\rm F}$ is specified by the Luttinger parameter $K_c(\neq 1)$ as $n(k) \propto \text{const.} - |k - k_{\rm F}|^{\theta}\text{sgn}(k - k_{\rm F})$ with a power-law exponent $\theta = (K_c - 1)^2/4K_c > 0$. Another important feature is spin-charge separation, due to which the one-electron spectral function $A(k, \omega)$ at $T = 0$ has two power-law divergences at energies $\omega = u_s(k - k_{\rm F})$ and $u_c(k - k_{\rm F})$, where u_s and u_c are the spin and charge velocities, respectively [4–6].

The central idea of the LL theory is the equivalence between the low-energy spectrum of the 1D system and that of the Luttinger model, which is a 1D solvable model of interacting electrons with linear dispersion [3]. The spin-charge separation is then precisely defined by the fact that the bosonized Hamiltonian for the Luttinger model fully separates into spin and charge parts. However, it does not mean that an extra electron added to the ground state of the Luttinger model is completely fractionalized into spin and charge excitations. Instead, the extra electron leaves behind itself an electron-like excitation with a velocity $v_{\rm F}$ in the spectral function. This was noted by Voit about two decades ago who considered a toy model with $\theta = 0$ but $u_s \neq u_c$, which is not a typical LL with $\theta > 0$, though [7]. Recently, a general proof of it has been given by us, including the clarification of the electron-like excitation for high energies and away from the Fermi point beyond the LL theory [8, 9].

It is, however, noted that the spectral function $A(k, \omega)$ of a 1D microscopic model (e.g., the 1D Hubbard model) generally differs from that of the Luttinger model even if the low-energy spectra of those models are equivalent to each other. For low energies and in the vicinity of $k_{\rm F}$, the difference in $A(k, \omega)$ is ascribed to the renormalization factor in a renormalization-group scheme. In the same scheme, the velocity $v_{\rm F}$ of an electron-like particle (renormalized

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electron or *pseudoelectron*) is not equal to the noninteracting Fermi velocity v_0 of the microscopic model but is, in general, renormalized.

In this paper, carefully discussing on the above renormalization effects, we study generic features of $A(k,\omega)$ of an interacting 1D electron system at $T=0$ for low energies and in the vicinity of $k_{\rm F}$. Even with the inclusion of those effects $A(k,\omega)$ is shown to have a singularity related to the pseudoelectron at $\omega = v_{\rm F}(k-k_{\rm F})$ as a function of ω with different prefactors above and below $v_{\rm F}(k-k_{\rm F})$. We obtain universal values for the power-law exponent and the prefactor ratio, which are uniquely determined by K_c. We find that the pseudoelectron velocity $v_{\rm F}$ is, in fact, given by the renormalized Fermi velocity at the LL fixed point and is equal to the slope of the lowest excitation spectrum near $k_{\rm F}$ in the system with large but finite size.

To illustrate our claim, we present some of exact results for the 1D Hubbard model at $n=0.59$ filling. It is noted that the overall structure of $A(k,\omega)$ for the 1D Hubbard model at this filling (or $n=0.6$) and $U=4.9$ t has been intensively studied by various methods in relation to the angle-resolved photoemission spectroscopy of the quasi-1D organic conductor TTF-TCNQ [10–15], although those studies are not concerned with the detailed structure of $A(k,\omega)$ for low energies and in the vicinity of $k_{\rm F}$. Here, we concentrate on it within the LL theory.

2 Renormalization-Group Scheme

Let us consider the spectral function $A(k_{\rm F}+q,\omega)$ for a 1D microscopic Hamiltonian $H[c^\dagger_{k,\sigma}, c_{k,\sigma}]$ ($\sigma=\pm$ labels the spin) and introduce a bandwidth cutoff Λ. By eliminating "fast" modes with $|k\mp k_{\rm F}|>\Lambda$ in a renormalization-group scheme, the spectral function can be written in the form

$$A(k_{\rm F}+q,\omega) = Z_{\rm F}\tilde{A}_+(k_{\rm F}+q,\omega)+\Delta A(k_{\rm F}+q,\omega), \qquad (1)$$

where the momentum q is limited by $|q|<\Lambda$ but the energy ω is allowed to take all values [16, 17]. Here, $\tilde{A}_+(k_{\rm F}+q,\omega)$ is the spectral function for the fixed-point Hamiltonian $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ ($\alpha=\pm$ distinguishes the right- and left-moving renormalized electrons) and $Z_{\rm F}$ is a renormalization factor that is introduced to satisfy the fermion anticommutation relation $\left\{\tilde{c}_{k,\alpha,\sigma}, \tilde{c}^\dagger_{k',\alpha',\sigma'}\right\} = \delta_{k,k'}\delta_{\alpha,\alpha'}\delta_{\sigma,\sigma'}$ in the fixed-point Hamiltonian. We then have the sum rule on $\tilde{A}_+(k_{\rm F}+q,\omega)$ as

$$\int_{-\infty}^{\infty}\tilde{A}_+(k_{\rm F}+q,\omega){\rm d}\omega = \left\langle\left\{\tilde{c}_{k_{\rm F}+q,+,\sigma}, \tilde{c}^\dagger_{k_{\rm F}+q,+,\sigma}\right\}\right\rangle = 1, \quad (2)$$

for any $|q|<\Lambda$. The last term $\Delta A(k_{\rm F}+q,\omega)$ represents an "incoherent" part in $A(k_{\rm F}+q,\omega)$. We do not expect that it gives an important contribution for low energies. Therefore, we do not consider it in the following.

It is, here, noted that (1) and (2) also hold in three-dimensional (3D) systems with suitable changes of the notation. In 3D, the fixed point is a Fermi liquid, where $Z_{\rm F}$ is finite for $\Lambda\to 0$ and $\tilde{A}_+(k_{\rm F}+q,\omega)$ corresponds to the quasiparticle's spectral function given by a Lorentzian. In 1D, on the other hand, the fixed point is a LL, where $Z_{\rm F}$ vanishes for $\Lambda\to 0$ as $Z_{\rm F}\propto(\Lambda/k_{\rm F})^\theta$ and $\tilde{A}_+(k_{\rm F}+q,\omega)$ has singularities as a function of ω.

If we consider the case of $q=0$, we can take the limit of $\Lambda\to 0$. Then, the dependence of $Z_{\rm F}$ on Λ is canceled by $\tilde{A}_+(k_{\rm F},\omega)\propto(v_0\Lambda)^{-\theta}|\omega|^{-1+\theta}$ in $A(k_{\rm F},\omega)$. In this paper, however, we are interested in the structure of low-energy excitations in $A(k_{\rm F}+q,\omega)$ and therefore we have to consider the case of a *finite* q. For a finite q, the limit of $\Lambda\to 0$ cannot be taken for $A(k_{\rm F}+q,\omega)$ because of the limitation of $|q|<\Lambda$, so that Λ must be scaled to $|q|$. This scaling argument leads to $Z_{\rm F}\propto(|q|/k_{\rm F})^\theta$, which is regarded as a finite number. By introducing a dimensionless function $f(x,y)$ through $\tilde{A}_+(k_{\rm F}+q,\omega) = f\left(\omega/v_0q, |q|/\Lambda\right)/v_0|q|$, we can write (1) with a finite q as

$$A(k_{\rm F}+q,\omega)\propto|q|^{-1+\theta}f\left(\omega/v_0q, |q|/\Lambda\right)/v_0k_{\rm F}^\theta. \qquad (3)$$

Then, by examining the characteristics of the function $f(x,y)$ irrespective of the argument y, we can find the universal behaviors of $A(k_{\rm F}+q,\omega)$ (see Section 4).

3 Finite-Size Spectrum

The LL fixed-point Hamiltonian $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ is composed of two branches of linear dispersion with a bandwidth cutoff Λ and is entirely characterized by a renormalized Fermi velocity $v_{\rm F}$, an intra-branch coupling g_4, and an inter-branch coupling g_2. On the other hand, the Luttinger model has an infinite bandwidth and momentum-dependent couplings with a momentum transfer cutoff Λ. There are four kinds of those couplings in general, which are conventionally denoted by $g_{4\parallel}(q)$, $g_{4\perp}(q)$, $g_{2\parallel}(q)$, and $g_{2\perp}(q)$. In this paper, we take $g_{4\parallel}(q)=0$ and $g_{2\parallel}(q)=g_{2\perp}(q)$ for all q together with $g_{4\perp}(0)=g_4$ and $g_{2\parallel}(0)=g_{2\perp}(0)=g_2$, so that the Luttinger model becomes equivalent to $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ except a cutoff scheme. The difference between two cutoff schemes for the bandwidth and the momentum transfer does not change physical results for low energies, but a mathematically rigorous treatment such as bosonization is possible for the latter scheme. Therefore, we can derive physical results for $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ from mathematically rigorous ones for the Luttinger model.

By using the above equivalence and the correspondence between the low-energy spectra of the fixed-point Hamiltonian $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ and the bare one $H[c^\dagger_{k,\sigma}, c_{k,\sigma}]$, we see that the spectrum of low-energy excitations in a 1D interacting electron system with large but finite size L (the number

of electrons in the ground state $N_G = 2\times$odd and $k_F \equiv \frac{\pi N_G}{2L}$) is given by

$$\Delta E = \frac{2\pi}{L}\sum_{\nu=s,c} u_\nu \left(\Delta_\nu^+ + \Delta_\nu^- + n_\nu^+ + n_\nu^-\right) + O(L^{-2}), \quad (4)$$

$$\Delta P = k_F J_c + \frac{2\pi}{L}\sum_{\nu=s,c}\left(\Delta_\nu^+ - \Delta_\nu^- + n_\nu^+ - n_\nu^-\right), \quad (5)$$

where $\Delta_c^\pm = \frac{(\delta N_c \pm J_c K_c)^2}{16K_c}$, $\Delta_s^\pm = \frac{(\delta N_s \pm J_s)^2}{16}$, $n_\nu^\pm$ is a nonnegative integer, δN_c, δN_s, J_c, and J_s are, respectively, quantum numbers related to total charge added into the system, spin, charge current, and spin current [3, 18, 19].

In (4) and (5), u_s, u_c, and K_c are given in terms of the LL fixed-point parameters v_F, g_4, and g_2 as $u_s = v_F - g_4/2\pi$, $u_c = \left[(v_F + g_4/2\pi)^2 - (g_2/\pi)^2\right]^{1/2}$, and $K_c = [(v_F + g_4/2\pi - g_2/\pi)/(v_F + g_4/2\pi + g_2/\pi)]^{1/2}$. We can solve those equations for v_F, g_4, and g_2. In particular, we find that the renormalized Fermi velocity v_F at the LL fixed point is given by

$$v_F = [u_s + (1+2\theta)u_c]/2, \quad (6)$$

where θ is related to K_c by $\theta = \frac{(1-K_c)^2}{4K_c}$ as already mentioned in Section 1.

To elucidate the physical meaning of (6), let us consider the spectrum in adding an electron with up spin in the vicinity of k_F. The corresponding quantum numbers are: $\delta N_c = J_c = \delta N_s = J_s = 1$. By substituting those quantum numbers into (4) and (5) and using (6), we obtain

$$\Delta E = (\pi/L)v_F + (2\pi/L)\sum_{\nu=s,c} u_\nu \left(n_\nu^+ + n_\nu^-\right) + O(L^{-2}), \quad (7)$$

$$\Delta P = k_F + \pi/L + (2\pi/L)\sum_{\nu=s,c}\left(n_\nu^+ - n_\nu^-\right). \quad (8)$$

Then, we see that v_F corresponds to the slope of the *lowest* excitation spectrum near k_F as $\lim_{L\to\infty} \Delta E/(\Delta P - k_F)$. Remember how the Fermi velocity v_0 is defined for a noninteracting system with a dispersion $\varepsilon_0(k)$. The lowest energy excitation near k_F is given by adding an electron to the unoccupied state with momentum $k_F + \pi/L$ ($k_F L/\pi$ is an odd integer), as sketched in Fig. 1. Then, the noninteracting Fermi velocity v_0 can be defined as $\lim_{L\to\infty} \Delta E/(\Delta P - k_F) = \lim_{L\to\infty}[\varepsilon_0(k_F + \pi/L) - \varepsilon_0(k_F)]/(\pi/L) = \partial\varepsilon_0(k)/\partial k|_{k=k_F}$. Because an interacting system has rigid Fermi points as well and the process of adding an extra electron into the system also obeys the Fermi statistics, we understand that (6) is a natural extension to the interacting case of v_0.

In Fig. 2, we have plotted v_F/v_0 as a function of the interaction U in units of $2\pi v_0$ together with u_s/v_0, u_c/v_0, and θ obtained by using the Bethe ansatz method for the 1D Hubbard model with the filling $n = 0.59$. Those values for $U = 4.9$ t will be used in obtaining the result in Fig. 3.

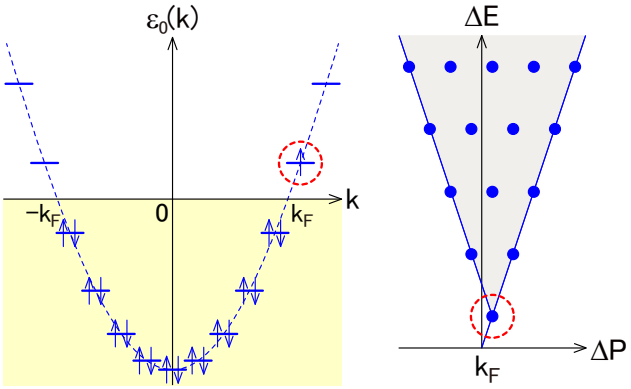

Fig. 1 Sketch of a process of adding an extra electron into the ground state of a noninteracting 1D system with finite size and the corresponding low-energy spectrum (color online)

4 One-Electron Spectral Function

The exact result of the Green's function of the Luttinger model is well known and it takes a simple form in real space and time [20, 21]. By using the physical equivalence between the Luttinger model and $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ discussed in the last section, we obtain the Green's function for the LL fixed-point Hamiltonian $\tilde{H}[\tilde{c}^\dagger_{k,\alpha,\sigma}, \tilde{c}_{k,\alpha,\sigma}]$ as

$$\tilde{G}_+(x,t) = \frac{\Lambda^{-\theta}}{2\pi}\frac{x - v_F t + i/\Lambda(t)}{x - v_F t + i\eta(t)}[x - u_s t + i/\Lambda(t)]^{-1/2} \times [x - u_c t + i/\Lambda(t)]^{-(1+\theta)/2} \times [x + u_c t - i/\Lambda(t)]^{-\theta/2}, \quad (9)$$

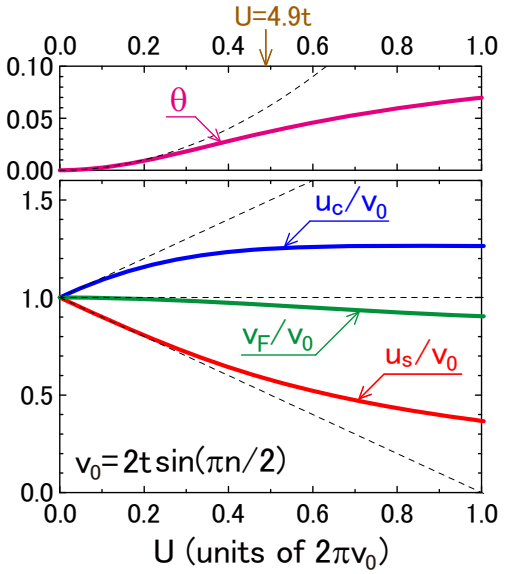

Fig. 2 Exact results of v_F/v_0, u_s/v_0, u_c/v_0, and θ as a function of the interaction U in units of $2\pi v_0 \approx 10\ t$ for the 1D Hubbard model with $n = 0.59$. The corresponding weak-coupling results are also shown by the broken curves (color online)

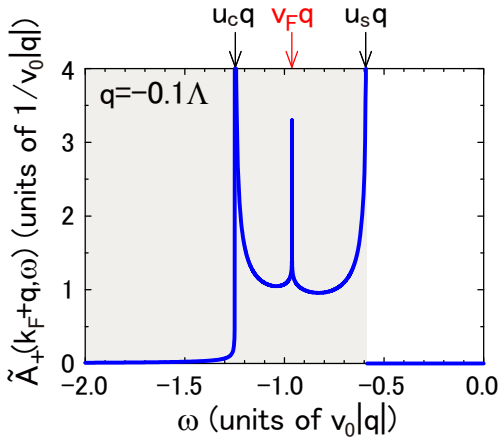

Fig. 3 Fixed-point spectral function $\tilde{A}_+(k_F+q,\omega)$ with $q=-0.1\Lambda$ for the 1D Hubbard model with $n=0.59$ and $U=4.9$ t (color online)

where $\eta(t)=\eta\,\mathrm{sgn}(t)$ with a positive infinitesimal η and $\Lambda(t)=\Lambda\,\mathrm{sgn}(t)$. This Green's function has the same form as that of the Luttinger model, but v_F, u_s, u_c, and θ are not parameters independent of one another. They are quantities that is derived from a 1D microscopic Hamiltonian as in Fig. 2.

The retarded Green's function $\tilde{G}^R_+(x,t)$ is derived from (9) by using $\tilde{G}^R_+(x,t)=\tilde{G}_+(x,t)+\tilde{G}^*_+(-x,-t)$ for $t>0$. The spectral function $\tilde{A}_+(k,\omega)\equiv-(1/\pi)\mathrm{Im}\tilde{G}^R_+(k,\omega)$ is then obtained by performing a double Fourier transform $\tilde{G}^R_+(k,\omega)=\int_0^\infty \mathrm{d}t\int_{-\infty}^\infty \mathrm{d}x\tilde{G}^R_+(x,t)\mathrm{e}^{\mathrm{i}\omega t-\mathrm{i}(k-k_F)x}$.

By using the above definition of $\tilde{A}_+(k,\omega)$, we can easily verify that the sum rule, (2), actually holds for (9). This is because (9) necessarily leads to $\tilde{G}^R_+(x,t=0^+)=-\mathrm{i}\delta(x)$, for which the presence of the second factor including v_F is essential. If we replace Λ by $1/\eta$ in (9), we have an approximate result of the Green's function without this factor, which is often cited in the literature as the Green's function obtained by the bosonization method. It is, however, noted that in the Luttinger model, taking $\Lambda=1/\eta\to\infty$ corresponds to neglecting the momentum dependence of the couplings, for which a mathematically rigorous formulation of bosonization to satisfy the fermion anticommutation relation cannot be possible unless $\theta=0$ [3]. In fact, a simple calculation leads to $\tilde{G}^R_+(x,t=0^+)|_{\Lambda=1/\eta}=-\mathrm{i}w(\theta)\delta(x)$ with $w(\theta)=\pi^{-1/2}\Gamma\,(1/2+\theta/2)\,/\,\Gamma\,(1+\theta/2)$. Because $w(\theta)<1$ for $\theta>0$, the replacement of Λ by $1/\eta$ results in a violation of the sum rule, (2), unless $\theta=0$.

Exact calculations of the spectral function of the Luttinger model with $\theta>0$ were first made with satisfying the sum rule by Schönhammer and Meden [22]. But their results are limited only for $v_F=u_s$. This is not the case in general for an interacting 1D system as clearly seen in Fig. 2. The calculations for a generic case were made by us by developing a new scheme to perform the double Fourier transform with use of the Feynman parameter trick, leading to a proof of the existence of an additional peak structure called the pseudoelectron excitation [8].

In Fig. 3, by using the same scheme, we have plotted the fixed-point spectral function $\tilde{A}_+(k_F+q,\omega)$ with $q=-0.1\Lambda$ for the 1D Hubbard model with $n=0.59$ and $U=4.9$ t. The peak-like cusp observed near the center is the pseudoelectron excitation. Because it is located at $\omega=v_Fq$, the pseudoelectron velocity is given by v_F, i.e., the renormalized Fermi velocity at the LL fixed point. It is, here, noted that since the function $f(x,y)$ introduced in Section 2 is defined by $\tilde{A}_+(k_F+q,\omega)=f\,(\omega/v_0q,|q|/\Lambda)\,/v_0|q|$, the plot in Fig. 3 corresponds to that of $f(-x,y)$ for $y=0.1$ as a function of x. We can show that for *any* value of y as well as $y=0.1$, the function $f(x,y)$ has divergences at $x=u_s/v_0,u_c/v_0$ and a peak-like cusp at $x=v_F/v_0$. On the other hand, a detailed description of those singularities is given by Eq. (4) in Ref. [8].

By using the above knowledge of $f(x,y)$ for (3), we can derive the universal behaviors of $A(k_F+q,\omega)\approx Z_F\tilde{A}_+(k_F+q,\omega)$ for low energies and in the vicinity of the Fermi point. For $|\omega-u_\nu q|\ll u_\nu|q|$ with $\nu=s,c$, we have

$$A(k_F+q,\omega)\approx a^\pm_\nu\frac{|q|^{-1+\theta}}{u_\nu k_F^\theta}\left|\frac{\omega-u_\nu q}{u_\nu q}\right|^{-\mu^{LL}_\nu}, \tag{10}$$

where the upper and lower signs of $a^\pm_\nu$ are taken for $|\omega|>u_\nu|q|$ and $|\omega|<u_\nu|q|$, respectively. The exponents are given by $\mu^{LL}_s=1/2-\theta$ and $\mu^{LL}_c=(1-\theta)/2$, a^+_s is a positive constant, $a^-_s=0$, and $a^\pm_c$ are positive constants with $a^+_c/a^-_c=\sin\pi\theta/2$, reproducing well-known results [4–6]. Our new result is a description of the pseudoelectron excitation including the renormalization effects. Namely, for $|\omega-v_Fq|\ll v_F|q|$, we find

$$A(k_F+q,\omega)\approx\frac{|q|^{-1+\theta}}{v_Fk_F^\theta}\left(b_0-b_\pm\left|\frac{\omega-v_Fq}{v_Fq}\right|^\theta\right), \tag{11}$$

where b_0 and $b_\pm$ are positive constants with $b_+/b_-=\cos\pi\theta$, and the upper and lower signs of $b_\pm$ are taken for $|\omega|>v_F|q|$ and $|\omega|<v_F|q|$, respectively. Therefore, the pseudoelectron excitation is characterized by the power-law exponent θ, the prefactor ratio $b_+/b_-=\cos\pi\theta$, and the velocity $v_F=[u_s+(1+2\theta)u_c]/2$.

5 Summary and Conclusion

We have studied generic features of the spectral function $A(k,\omega)$ of an interacting 1D electron system at $T=0$ for low energies and in the vicinity of the Fermi point. By a scaling argument based on a renormalization-group scheme, we have shown that the pseudoelectron excitation in $A(k,\omega)$ is described by (11) to find that the universal exponent is

equal to θ and the universal ratio of the prefactors is given by $b_+/b_- = \cos\pi\theta$ with $\theta = (K_c{-}1)^2/4K_c$. Then, in addition to the two power-law divergences at $\omega = u_s(k-k_{\rm F})$ and $\omega = u_c(k-k_{\rm F})$, $A(k,\omega)$ has an asymmetric peak-like cusp characterized by those universal quantities at $\omega = v_{\rm F}(k{-}k_{\rm F})$.

The pseudoelectron corresponds to the renormalized electron in the renormalization-group language and its velocity is given by the renormalized Fermi velocity at the LL fixed point, an important quantity to determine the position of the peak-like cusp. We have found that the pseudoelectron velocity is given by $v_{\rm F} = [u_s{+}(1{+}2\theta)u_c]/2$ and is equal to the slope of the lowest excitation spectrum near $k_{\rm F}$ in the system with large but finite size. For the 1D Hubbard model at $n = 0.59$ filling, we have obtained explicit values of $v_{\rm F}$ as a function of the interaction U.

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